DIFFRACTION OF A STRONG SHOCK WAVE AT A THIN WEDGE

I. V. Simonov

UDC 532.593

The diffraction of a strong shock wave at a wedge is investigated on the assumption of a small difference between the properties of the medium and the wedge. The wedge angle and its location relative to the wave front are arbitrary.

At the high pressures and temperatures occurring behind a strong shock ($P \sim 10^6$ atm) it is reasonable in many cases in a theoretical treatment to neglect the resistance of the material and to describe its state by comparatively simple models of the medium, e.g., the perfect fluid model. Comparison of the results of such a treatment with experiment yields boundaries for the application of the model and a direct indication of the effect of the neglected factors.

The problem addressed here reduces to a Hilbert problem. It turns out that the condition for the existence of a solution to the Hilbert problem in a class of functions having a zero of at least second order at infinity is identically the condition for stability of the shock wave in a homogeneous medium [1, 2].

1. A plane strong stationary shock wave, moving through a homogeneous unstressed medium with speed D_0, meets a wedge embedded in the medium at time $t=0$. The faces of the wedge make angles α_1 and α_2 with the surface of the wave front; P_0, V_0, and U_0 are, respectively, the pressure, specific volume, and mass velocity behind the incident shock, 0, V, 0 and 0, V', 0 being, respectively, their values ahead of the front in the medium and in the wedge; $V_*^0 = V_*^\circ(P)$ and $V_* = V_*(P)$ are the normal shock adiabat equations for the material of the medium and wedge, giving the specific volumes as a function of pressure.

The initial densities and the behavior of the materials of the medium and wedge when shock loaded are not very different. We introduce the parameter

$$\varepsilon = \max \frac{|V_*^\circ(P) - V_*(P)|}{V_*^\circ(P)}, \quad 0 \leqslant P \leqslant P' \quad (P' > P_0)$$

Then for $\varepsilon = 0$ the shock wave does not see the wedge; and for $\varepsilon \ll 1$ we address the problem of determining the small perturbations resulting from diffraction of all the quantities, in a linear approximation. Here we still consider that the order of smallness of the pressure perturbations is less than that of material strength properties behind the shock, so that effects associated with the strength can be neglected. We note that as the shock strength increases, so does the role of the thermal components of the pressure and internal energy of the material behind the shock (in the limit the solid behaves as a gas) [3]. Accordingly, the role of the strength properties diminishes. Therefore, the above approximation makes sense.

It is reasonable to postulate that the speed of sound in the medium and in the wedge behind the shock differ by only a small amount. Hereafter we shall neglect this difference (and therefore drop terms of higher order of smallness in the corresponding equations) and assume the sound speed everywhere behind the front to be c.

We postulate that the inequality $D_0 - U_0 < c$ is satisfied. Then for $\alpha_{1,2} < \alpha_*$, where α_* is the limiting angle, whose value will be determined below from the condition of regular refraction of the shock at the faces of the wedge, the picture of the diffraction is as shown in Fig. 1. Then, in the vicinity of the points of intersection of the shock front with the wedge faces, a triangular wave configuration is formed with incident and refracted shocks and reflected sound waves and flow zones with constant parameters. An unsteady disturbance propa-

---

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 107-114, November-December, 1971. Original article submitted May 17, 1971.

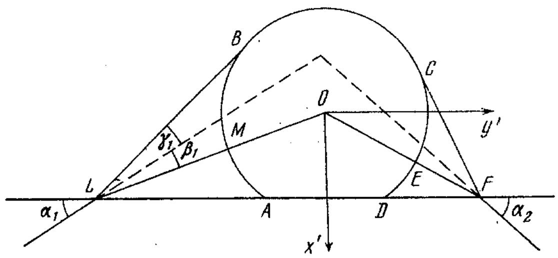

Fig. 1

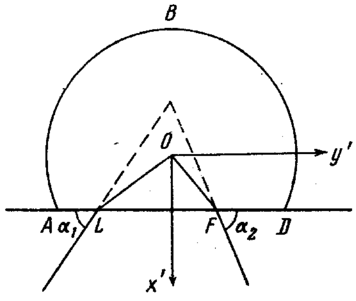

Fig. 2

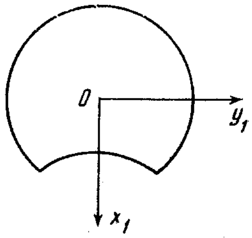

Fig. 3

gates from the wedge center displaced by the flow, bounded by the arc of the Mach circle ABCD and the attached fronts of reflected sound waves, and by the shock wave section AD.

For $\alpha_{1,2} > \alpha_*$ in the vicinity of the points of intersection of the shock front with the wedge faces, there is nonregular refraction (Fig. 2). There is only one region of unsteady flow which spans the sections of the shock wave outside the wedge. For $\alpha_1 > \alpha_*$, $\alpha_2 < \alpha_*$ and vice versa the picture of the diffraction is clearly a combination of the above cases.

We introduce a system of coordinates x'y' fixed at the moving center of the wedge, and the following notation for quantities in the perturbed region: P – pressure, $\mathbf{U} - \mathbf{U}_0$ – mass velocity, V and V'_1 – specific volumes in the medium and in the wedge.

Now we write the shock adiabat equation in the form

$$V_*(P) = V_*^\circ(P)[1 + \varepsilon v_*(P)] \tag{1.1}$$

and we approximate to the unknowns in the form

$$P \approx P_0 + \varepsilon p', \qquad \mathbf{U} - \mathbf{U}_0 \approx (\varepsilon u', \varepsilon w')$$

The perturbed quantities p', u', and w' satisfy the ordinary linearized equations of two-dimensional flow of an ideal compressible fluid.

The problem considered is one where the functions p', u', and w' are homogeneous functions of the coordinates and the time of zero measurement. We introduce the dimensionless and similarity variables

$$p = V_0 p' / c^2, \ u = u' / c, \ w = w' / c, \ x = x' / ct, \ y = y' / ct$$

The equations for p, u, and w have the form

$$Dp = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial y}, \ Du = \frac{\partial p}{\partial x}, \ Dw = \frac{\partial p}{\partial y} \quad \left(D = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right) \tag{1.2}$$

In order to derive the conditions in the perturbed section of the shock front, we use Eq. (1.1) to represent V' and V_1, V_1' at the front, and the equation of the front in the form

$$\begin{gathered} V' = V_*(0) = V_*^\circ(1 + \varepsilon v') \\ V_1 \approx V_0(1 - \varepsilon j_0 p'), \ V_1' \approx V_0[1 + \varepsilon(v_0 - j_0 p')] \quad \text{for} \quad x = k \\ x \approx k + \varepsilon f(y) \\ \left(v' = v_*(0), \ j_0 = -\frac{1}{V_0}\left(\frac{dV_*^\circ}{dP}\right)_{P=P_0}, \ v_0 = v_*(P_0)\right) \end{gathered} \tag{1.3}$$

Then these conditions take the form

in the medium

$$p = A\,(f - yf'),\ u = Bp,\ w = -Mf' \text{ for } x = k$$

in the wedge

$$p = A\,(f - yf') + A_1,\ u = Bp + B_1,\ w = -Mf' \text{ for } x = k$$

$$A = \frac{2\varkappa M}{1-j},\ A_1 = kM\frac{(2\varkappa - 1)\,v' - \varkappa v_0}{(1-j)(1-\varkappa)},\ B = \frac{1+j}{2k}$$

$$B_1 = M\frac{v' - \varkappa v_0}{2\,(1-\varkappa)} \quad \left(\varkappa = \frac{V_0}{V},\ M = \frac{U_0}{c},\ k = \frac{D_0 - U_0}{c},\ j = \frac{k^2c^2j_0}{V_0}\right)$$

We reduce the problem to a search for the single function p. The solution for p(x, y) is constructed differently in the regions $r<1$ and $r>1$ ($r^2=x^2+y^2$), since the type of equation that p(x, y) satisfies is elliptic in the first region and hyperbolic in the second. We join these solutions at the boundary $r=1$ to make them continuous [4]; this is done by satisfying the conditions of dynamic and kinematic compatibility.

From the condition for continuity of pressure and normal velocity component on the line of the contact discontinuity LOF, we have that the function p(x, y) and its first derivatives are continuous on LOF. This is a consequence of linearizing the problem.

2. In the case $\alpha_{1,2} < \alpha_*$ we examine the flow in regions LBM and LMA (Fig. 1). To determine the angles γ_1 and β_1, which determine the position of the sound front LB and of the wedge face displaced by the flow, we write down the relations

$$\frac{D_0}{\sin\alpha_1} = \frac{c - U_0\cos(\alpha_1 + \gamma_1)}{\sin\gamma_1} = \frac{U_0\cos(\alpha_1 - \beta_1)}{\sin\beta_1} \tag{2.1}$$

which derive from the condition that, in the coordinate system with the front LB and the point of intersection of the fronts L fixed, the lines of the fronts are fixed.

We obtain

$$\gamma_1 = 2\,\mathrm{arc\,tg}\,\frac{k_0 - M\sin^2\alpha_1 - \sqrt{(k_0 - M\sin^2\alpha_1)^2 + 1/4\,M^2\sin^2 2\alpha_1 - \sin^2\alpha_1}}{1/2\,M\sin 2\alpha_1 + \sin\alpha_1}$$

$$\beta_1 = \mathrm{arctg}\,[(1 - \varkappa)\,\mathrm{tg}\,\alpha_1 / (1 + \varkappa\,\mathrm{tg}\,\alpha_1)] \qquad (k_0 = D_0 / c_0) \tag{2.2}$$

Here one of the two solutions of the first equation of Eq. (2.1) is chosen as satisfying the physical sense of the problem.

We seek a constant solution in the regions LBM and LMA. Let u_n^1 be the discontinuity in velocity perturbation in passing through LB; u_1 and w_1 the perturbed velocity components in region LBM; and p_1 the perturbed pressure.

Adding the condition at an acoustic discontinuity

$$p_1 = u_n^{\,1} \tag{2.3}$$

to the conditions (1.4) at the front, and also the condition for continuity of the normal component of velocity perturbation at the contact discontinuity LO

$$-u_n^{\,1}m = u_1 + w_1\,\mathrm{tg}\,(\alpha_1 - \beta_1) \qquad (m = \cos(\gamma_1 + \beta_1)\,/\cos(\alpha_1 - \beta_1))$$

and taking into account that

$$f\,(y_L) = 0$$

we obtain a closed system of linear equations to determine the constant parameters u_n^1, p_1, w_1, u_1, and f'. Then

$$p_1 = \frac{kM}{1-\varkappa}\,\frac{\varkappa v_0 - v' + \varkappa\,(\varkappa v_0 + (1 - 2\varkappa)\,v')\,\mathrm{tg}^2\alpha_1}{2km + 1 + j - \varkappa\,(1 - j)\,\mathrm{tg}^2\alpha_1} \tag{2.4}$$

The remaining quantities are determined simply in terms of p_1 from Eqs. (1.4) and (2.3).

The parameters of the constant flow zones ECF and DEF are calculated via similar formulas. We shall denote these quantities by the same letters with subscript 2.

We note that at the displaced contact boundary LOF there is a tangential discontinuity in the velocity vector perturbation.

We determine the limiting angle α_* from the condition that in the x'y' coordinate system the velocity of the point of intersection of the fronts is equal to the sound speed c:

$$\alpha_* = \arcsin (k_0 / \sqrt{k_1^2 + k_0^2}) \qquad (k_1 = \sqrt{1 - k^2})$$

It should be noted that the numerator in the expression for p_1 can generally go to zero for a certain value α_1. It takes a minimum value for $\alpha_1 = \alpha_*$, and it is positive for small enough α_1. For a solution to exist for all $0 \le \alpha \le \alpha_*$ we require that the condition

$$\varkappa k_1^2 (1 + j) - k^2 (1 - j) > 0 \tag{2.5}$$

be satisfied.

Here we have simultaneously solved the problem of regular refraction of a strong plane shock wave at the interface of two slightly different semi-spaces. For $\alpha_1 \to 0$ the solution of the problem of a normally incident shock is obtained.

3. From the condition of pressure continuity in passing through the arc of the Mach circle, Eqs. (1.4), and Eq. (1.3), we find the boundary condition for the normal and tangential derivatives of the pressure

$$a \partial p / \partial n + b \partial p / \partial s = d$$

Here n is the external normal and s is the tangent in going around ABCD positively (Fig. 1) (ABD in Fig. 2)

$$a = 1, \quad b = B_2 y^{-1} - (B + k) k_1^{-2} y \quad \text{on} \quad AD$$
$$a = 0; \quad b = 1 \quad \text{on} \quad ABCD \ (ABD)$$
$$d = p_1 \delta (\theta - \theta_1) - p_2 \delta (\theta - \theta_2) \qquad \text{for} \ \alpha_{1,2} < \alpha_*$$
$$d = (B_1 k_1^{-2} y + A_1 B_2 y^{-1}) (\delta (y - y_1) - \delta (y - y_2)) \quad \text{for} \quad \alpha_{1,2} > \alpha_*$$
$$B_2 = kM / k_1^2 A, \qquad \delta (\theta) - \text{the Dirac delta function}$$
$$\theta_1 = \theta_B = \pi + \alpha_1 + \gamma_1, \ \theta_2 = \theta_C = \pi - \alpha_2 - \gamma_2, \ y_1 = y_L =$$
$$= -k\varkappa^{-1} \operatorname{ctg} \alpha_1, \ y_2 = y_F = k\varkappa^{-1} \operatorname{ctg} \alpha_2, \ \theta = \operatorname{arc\,tg}(y / x)$$

For x=k we obtain from Eq. (1.4)

$$\frac{1}{y} \frac{\partial p}{\partial y} = \frac{A}{M} \frac{\partial w}{\partial y} \qquad (\alpha_{1,2} < \alpha_*)$$
$$\frac{1}{y} \frac{\partial p}{\partial y} = \frac{A}{M} \frac{\partial w}{\partial y} + \frac{A_1}{y} (\delta (y - y_1) - \delta (y - y_2)) \qquad (\alpha_{1,2} > \alpha_*)$$

Integrating the latter along the perturbed section of the shock wave AD, we obtain the condition for smooth junction of the front

$$\begin{gathered} \int_{AD} \frac{1}{y} \frac{\partial p}{\partial y} dy = A (w_2 - w_1) \quad \text{for} \quad \alpha_{1,2} < \alpha_* \\ \int_{AD} \frac{1}{y} \frac{\partial p}{\partial y} dy = A_1 \left(\frac{1}{y_1} - \frac{1}{y_2} \right) \quad \text{for} \quad \alpha_{1,2} > \alpha_* \end{gathered} \tag{3.1}$$

Following the method of functionally invariant Smirnov-Sobolev solutions [4], we transfer the problem to the complex variable plane

$$z = x_1 + i y_1 = (r^{-1} - \sqrt{r^{-2} - 1}) \exp (i\theta)$$

and put

$$p = \operatorname{Im} P (z)$$

The region of unsteady flow in the complex plane z corresponds to that shown in Fig. 3. The equations of the circular arc have the form

$$| z | = 1, \ 2 | z | \cos\theta = k (1 + | z |^2)$$

We perform conformal mapping of the interior of the figure to the upper half-plane by means of the transformation

$$\zeta = \xi + i\eta = \frac{i}{k_1}\frac{2z - k(1+z^2)}{z^2-1}$$

Here the arc corresponding to the section AD of the shock wave transforms to the interval $-1 < \xi < 1$; the points B and C lie outside this interval, and the points L and F lie inside it.

We introduce the function

$$F^+(\zeta) = \frac{\partial p}{\partial \eta} + i\frac{\partial p}{\partial \xi} = \frac{dP}{d\zeta}$$

which is analytical in the upper half-plane of ζ. On the real axis $F^+(\zeta)$ satisfies the condition

$$a\frac{\partial p}{\partial \eta} + b\frac{\partial p}{\partial \xi} = d \tag{3.2}$$

Here

$$a = \xi\sqrt{1-\xi^2}, \qquad b = B\xi^2 - B_2 \qquad (|\xi| < 1)$$
$$a = 0, \qquad b = B - B_2 \qquad (|\xi| > 1)$$
$$d = (B - B_2)(p_1\delta(\xi - \xi_1) - p_2\delta(\xi - \xi_2)) \qquad (\alpha_{1,2} < \alpha_*)$$
$$d = (B_1\xi^2 + A_1B_2)(\delta(\xi - \xi_3) - \delta(\xi - \xi_4)) \qquad (\alpha_{1,2} > \alpha_*)$$
$$\xi_1 = \xi_B = k_1^{-1}(\operatorname{cosec}\theta_1 - k\operatorname{ctg}\theta_1), \quad \xi_2 = \xi_C = k_1^{-1}(\operatorname{cosec}\theta_2 - k\operatorname{ctg}\theta_2)$$
$$\xi_3 = \xi_L = -\varkappa^{-1}\operatorname{ctg}\alpha_1, \qquad \xi_4 = \xi_F = \varkappa^{-1}\operatorname{ctg}\alpha_2$$

We have thus formulated an inhomogeneous Hilbert problem with continuous coefficients. We seek a solution of it in the class of functions that have a zero of at least second order at infinity (this stems from the requirement that the function p(z) be bounded as $z \to \infty$).

The index of the problem is unity if

$$0 < k^2(1-j) < \varkappa k_1^2(1+j) \tag{3.3}$$

and zero if

$$k^2(1-j) > \varkappa k_1^2(1+j) \quad \text{or} \quad j > 1 \tag{3.4}$$

In case (3.4) there is no solution in this class of functions. To check this, it is enough to write the solution for this case in the standard formula (14.22') of [5], p. 265. We note that the condition $k^2(1-j) > \varkappa k_1^2(1+j)$ is the same as requirement (2.5).

The condition for the existence of a solution of Eq. (3.3) is equivalent to the condition for stability of a plane stationary shock wave in a homogeneous medium [1, 2]. This is an interesting point, which confirms the need for the limitation (3.3) on the properties of the medium for a stable shock to exist in it.

From now on we consider Eq. (3.3) to be satisfied. We find the regularizing factor $q(\xi)$ of the function $b(\xi) + ia(\xi)$ by continuing the function $B\xi^2 - B_2 + i\xi\sqrt{1-\xi^2}$ over the whole real axis:

$$q(\xi) = \begin{cases} (B\xi^2 - B_2 + \xi\sqrt{\xi^2-1})/(B - B_2), & \xi > 1 \\ (B\xi^2 - B_2 - \xi\sqrt{\xi^2-1})/(B - B_2), & \xi < -1 \\ 1, & |\xi| < 1 \end{cases}$$

Multiplying both sides of Eq. (3.2) by $q(\xi)$, we bring it to the form

$$\operatorname{Im}[F^+(\xi)/\Phi^+(\xi)] = q(\xi)\,d(\xi) \tag{3.5}$$

where $\Phi^+(\xi)$ is the boundary value of the analytic functions defined in the upper half-plane

$$\Phi^+(\zeta) = \frac{1}{B\zeta^2 - B_2 + i\zeta\sqrt{1-\zeta^2}}$$

Here the radical must be formed so that it takes positive values for $-1 < \xi < 1$, $\eta = 0$.

For condition (3.3), $\Phi^+(\xi)$ has no singularities in the upper half-plane, including the real axis.

Extrapolating the boundary condition (3.5) into the complex plane and taking into account that the function $F^+(\zeta)/\Phi^+(\zeta)$ must be regular at infinity, we have

$$F^+(\zeta) = \Phi^+(\zeta)\,[\Psi^+(\zeta) + C_0] \tag{3.6}$$

Here

$$\psi^+(\zeta) = \begin{cases} \dfrac{1}{\pi}\left(\dfrac{p_1}{\Phi^+(\xi_1)(\xi_1-\zeta)} - \dfrac{p_2}{\Phi^+(\xi_2)(\xi_2-\zeta)}\right), & \alpha_{1,2} < \alpha_* \\ \dfrac{1}{\pi}\left(\dfrac{B_1\xi_4^2 + A_1B_2}{\xi_4-\zeta} - \dfrac{B_1\xi_3^2 + A_1B_2}{\xi_3-\zeta}\right), & \alpha_{1,2} > \alpha_* \end{cases}$$

The real constant C_0 is determined from condition (3.1).

The transformation to the original similarity coordinates is accomplished via the formula

$$\zeta = \frac{y(1-kx) + i(k-x)\sqrt{1-x^2-y^2}}{k_1(1-x^2)}$$

The pressure is determined from the formula

$$p(\xi,\eta) = \operatorname{Im}\int_{-1}^{\zeta}\left(\frac{\partial p}{\partial \eta} + i\frac{\partial p}{\partial \xi}\right)d\zeta + p_A$$

Here

$$p_A = p_1 \quad \text{for} \quad \alpha_{1,2} < \alpha_*, \quad p_A = 0 \quad \text{for} \quad \alpha_{1,2} > \alpha_*$$

In the case $\alpha_{1,2} > \alpha_*$, the pressure (and also f', u, and w) has a logarithmic singularity at the points L and F. A similar singularity appears in the case of a subsonic incident shock wave on a thin wedge in the Lighthill problem [6, 7]. We note that this singularity vanishes for $\alpha_1 = \alpha_*$ and $\alpha_1 = 0$. Here the solution for $\alpha_1 > \alpha_*$ goes over continuously to that for $\alpha_1 < \alpha_*$.

Along the curved section AD of the shock wave the pressure distribution has the form

$$p(y) = \int_{-1}^{y/k_1}\left(\frac{\partial p}{\partial \xi}\right)_{\eta=0} d\xi + p_A, \qquad \left(\frac{\partial p}{\partial \xi}\right)_{\eta=0} = \frac{\xi\sqrt{1-\xi^2}}{(B\xi^2 - B_2)^2 + \xi^2(1-\xi^2)} \times$$
$$\times\left[\frac{p_1}{\pi\Phi^+(\xi_3)(\xi-\xi_3)} - \frac{p_2}{\pi\Phi^+(\xi_4)(\xi-\xi_4)} + C_0\right] \qquad (\alpha_{1,2} < \alpha_*) \tag{3.7}$$
$$\left(\frac{\partial p}{\partial \xi}\right)_{\eta=0} = \frac{1}{(B\xi^2 - B_2)^2 + \xi^2(1-\xi^2)}\Big[(B\xi^2 - B_2)(\varphi(\xi_2)\delta(\xi-\xi_2) - \varphi(\xi_1)\delta(\xi-\xi_1)) -$$
$$- \xi\sqrt{1-\xi^2}\left(\frac{\varphi(\xi_2)}{\pi(\xi_2-\xi)} - \frac{\varphi(\xi_1)}{\pi(\xi_1-\xi)} + C_0\right)\Big], \quad \varphi = B_1\xi^2 + A_1B_2 \qquad (\alpha_{1,2} > \alpha_*)$$

Here we should take the integral in Eq. (3.7) to have the principal value.

The remaining functions are determined in closed form in terms of p. For example, the shape of the curved portion of the shock wave is calculated from the formula

$$f(y) = \frac{1}{A}\left[p(y) + y\left(\frac{p_A \operatorname{tg}\alpha_1}{k+M} - \frac{1}{k_1}\int_{-1}^{y/k_1}\frac{1}{\xi}\left(\frac{\partial p}{\partial \xi}\right)_{\eta=0} d\xi\right)\right]$$

The functions u and w are determined from Eq. (1.2):

$$u(r,\theta) = \int_{r_0(\theta)}^{r}\frac{1}{\rho}\frac{\partial p}{\partial x}d\rho + u_0(\theta), \quad w(r,\theta) = \int_{r_0(\theta)}^{r}\frac{1}{\rho}\frac{\partial p}{\partial y}d\rho + w_0(\theta)$$

Here the integration is performed along the radius

$$r_0(\theta_0 < \theta < 2\pi - \theta_0) = 1, \quad r_0(-\theta_0 < \theta < \theta_0) = k/\cos\theta,$$
$$\theta_0 = \operatorname{arctg}(k_1/k)$$

and $u_0(\theta)$ and $w_0(\theta)$ are the values of u and w on the boundary of ABCD (ABD).

We find the shape of the perturbed contact boundary

$$\psi(s) = -s \int_{s_0}^{s} \rho^{-2} u_n(\rho)\, d\rho$$

Here s is the coordinate along LO, $s_0 = k/\cos(\alpha_1 - \beta_1)$ for $\alpha_1 < \alpha_*$, $s_0 = 1$ for $\alpha_1 > \alpha_*$, $\psi(s)$ is the displacement of the contact boundary along the normal to LO, and

$$u_n(s) = u(s) \cos(\alpha_1 - \beta_1) + w(s) \sin(\alpha_1 - \beta_1)$$

The shape of the FO boundary can be determined analogously.

We note that in the case $\alpha_1 < \alpha_*$, $\alpha_2 > \alpha_*$ the solution is constructed simply by combining the solutions with regular and nonregular refractions. In the special case, putting $\alpha_1 = 0$, $\alpha_2 = \pi/2$, we can obtain diffraction at a right angle.

In conclusion we note that there is no theory for nonregular refraction. The results obtained in this paper may be of interest from this viewpoint.

The author thanks N. V. Zvolinskii and L. M. Flitman for discussion of the work.

LITERATURE CITED

1. C. P. D'yakov, "Stability of shock waves," Zh. Éksper. i Teor. Fiz., 27, No. 3 (1954).
2. S. V. Iordanskii, "Stability of a plane stationary shock wave," Prikl. Matem. i Mekhan., 21, No. 4 (1957).
3. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High Temperature Hydrodynamic Phenomena [in Russian], Nauka, Moscow (1966).
4. F. Frank and R. Mises, Differential and Integral Equations of Mathematical Physics [in Russian], ONTI, Gostekhizdat, Moscow–Leningrad (1937).
5. F. D. Gakhov, Boundary Problems [in Russian], Fizmatgiz, Moscow (1963).
6. M. J. Lighthill, "The diffraction of blast waves," Proc. Roy. Soc., A198, No. 1055 (1949).
7. K. A. Bezhanov, "Theory of shock wave diffraction," Prikl. Matem. i Mekhan., 24, No. 4 (1960).